The Neutrinos and Force Constants of Geometric Force-Field structures wherein a *First-Principle Distorted-Geometry-Construct* mimics Mass(*hydrostatic stress*) and Electric Charge(*shear stress*)

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Abstract

It is shown in the present work that the distorted-space model of matter can describe conventional force-constants and the associated geometric force-field structures. We use the verbiage "distorted" to communicate the concept of "energetic warping" to distinguish "spatial warping" from "classical matter warping," although the concept of "matter" is in fact, in the present context, the "geometric distortion energy" of the spatial manifold itself without a classical "matter stress-energy source." The "distorted-geometry" structures exhibit non-Newtonian features wherein the hole or core-region fields of the structures are energetically-repulsive (negative pressure), do not behave functionally in an r^{-4} manner and terminate at zero at the radial origin (no singularity). Near the core of the distortion the magnetic fields dominate the energy-densities of the structures thereby departing from classical particle-structure descriptions. Geometrically-defined force constants are constituted from the tensor energy-density (pressure) elements. Black-body and gray-body radiation-emission and structural modeling lead to a description of transition dynamics and photonic entities. The ubiquitous electron-neutrino, a geometric-entity and one of a triad, (the **tauon-neutrino**, the **muon-neutrino**, and the **electron-neutrino**), is illustrated at a mass-energy = 0.8118 eV and an electric-charge = $1.01\ (10)^{-11}$. The "core" **magnetic**

energy-density radius, near the zero of the **magnetic** energy-density distribution, is = $1.924(10)^{-15}$ meters and the "core" **electric** energy-density radius is = $1.808(10)^{-31}$ meters illustrating the vanishingly small electric-charge feature which would satisfy a present-day, Q = 0, description of said neutrino.

1. Introduction

A comprehensive description or model of "matter in the universe" at the fundamental level, which improves on the "Newtonian r^{-4} gravitational force model" (mathematically has an infinity at r = 0 labelled a singularity), is proposed. Matter and force concepts are to be replaced by more "ab initio" or "first principle" energy-producing, Geometry-based, structural modeling concepts. We have developed a description of matter as a "distorted or warped, non-flat and therefore energy-dense geometric" structure, a geometric-mimic of matter's defining physical characteristics, formulated from a solution of Riemann's geometric equations with a composite Electromagnetic (EM) and a Gravitational coupling-constant (see Supplementary Information below). **Physical mass** is a measure of the **total** (**hydrostatic**) distorted-geometry spatial-energy and **electric-charge (shear stress)** is a measure of the distorted-geometry (coupled static **Electromagnetic**) spatial-energy.

The geometric equations describing the "local region in examination," a region extending to the "local-origin- r = 0" since no "matter stress-energy-elements" are present, are the 3-dimensional, static, spatial, "Riemannian geometric-curvature equations ($1/r^2$ or $1/r$ by definition)" [1, 2(pp 264)]. By expressing these geometric-curvature elements (using a spatial 3-dimensional (stationary) spherical-coordinate system with indices (index 1 ≡ radial index r, index 2 ≡ azimuthal index θ and index 3 = azimuthal index ϕ) in "energy-density" form by applying a "stress-energy coupling-constant" (classically a gravitational coupling-constant has been used), we produce a geometrically-based, 3-dimensional, spatial version of the Riemannian "energy-density (matter)" tensors (Joule/meter^3)] as a description of the "stressed" region of space.

The model is essentially the "Curved empty space as the building material of the physical world" supposition of Clifford [3] in 1876 and is the conceptual basis for this "distorted-geometry" modeling. Such a geometric description of localized warping or distorting (a distension of the "curvature of the spacetime manifold") would seem to constitute a "first-principle" model of the universe achieved only by ascribing to the spatially-distorted (warped) region a "material," or "distortable", characteristic.

This work describes a geometrically warped (or distorted) region of space as an energy-mimic of matter and in a particular form as a "theoretically-posited intermediate structure (a 3rd -phase or intermediate stage formed after the starting stage and before the final-stage structure)", that is, a transitory fundamental-particle

structural-form such as the W-boson (Mediator particles or Force carriers) in the beta-decay-process for example.

Quoting the "Force carrier" reference, "In quantum field theory, force carriers or messenger particles or intermediate particles are particles that give rise to forces between other particles. These particles are bundles of energy (quanta) of a particular kind of "field" (physics definition). There is one kind of field for every type of elementary particle. For instance, there is an electromagnetic field whose quanta are photons. The concept is especially important in particle physics where the force carrier particles that mediate the electromagnetic, weak, and strong interactions are called gauge bosons."

The present "geometry based" model, with its "distorted material-like" description of "matter," ("a solution" of Riemann's geometrical, second-order, non-linear differential equations) departs from the "force carrier" construct and exhibits the physical characteristics of all of the four "**force-fields**", gravity, electromagnetism, the weak-force, and the strong-force.

A black-body emission-based energy-transition dynamics to describe the photonic transition process is postulated and presented.

At The International Congress for Logic, Methodology, and Philosophy of Science in 1960, Wheeler [4] began by quoting William Kingdon Clifford's [3] "Space-Theory of Matter" of 1876 and stated "The vision of Clifford and Einstein can be summarized in a single phrase, 'a geometrodynamical universe': a world whose properties are described by geometry, and a geometry whose curvature changes with time – a dynamical geometry."

Although these authors were trying to include motion and therefore time dependence in their considerations, the present work and modeling is restricted to local-regions which can be modelled with time-independent, or static-only, spatially-dependent metrics and energy-density tensor entities.

Similar "static-modeling" was accomplished by Schwarzschild [5](also see Tolman [2, p245]) and see https://en.wikipedia.org/wiki/Schwarzschild_metric which is quoted here in this regard: "In Einstein's theory of general relativity, the Schwarzschild metric (also known as the Schwarzschild solution) is an exact solution to the Einstein field equations that describes the gravitational field outside a spherical mass, on the assumption that the electric charge of the mass, angular momentum of the mass, and universal cosmological constant are all zero. The solution is a useful approximation for describing slowly rotating astronomical objects such as many stars and planets, including Earth and the Sun. It was found by Karl Schwarzschild in 1916, and around the same time independently by Johannes Droste, who published his much more complete and modern-looking discussion only four months after Schwarzschild.

According to Birkhoff's theorem , the Schwarzschild metric is the most general spherically symmetric vacuum solution of the Einstein field equations.

A Schwarzschild black hole or static black hole is a black hole that has neither electric charge nor angular momentum. A Schwarzschild black hole is described by the Schwarzschild metric, and cannot be distinguished from any other Schwarzschild black hole except by its mass."

Clifford's work, at least his conceptual"Curved empty space as the building material of the physical world" supposition, or the "Space-theory of Matter," predated Einstein's concepts and work on General Relativity by 40 years.

Additional work in this field continues, some of which is cited in references [6-11]. The present treatment or model departs from these cited "constructional methods" in that we do not constrain the warped-geometry descriptions to only gravitational-coupling-constant (G/c^4) produced structures.

The task or undertaking of the present work can be stated as follows; "{1} Construct the geometric description (equations) of a static, warped (distorted), spherically-symmetric, localized region of 3-dimensional space satisfying, or being characterized by, a material-like quality expressed as an "equation-of-state". {2} Solve said geometric equations and utilize the solutions, if possible, to mimic the physical descriptors (characteristics) of matter".

Riemann [1] has provided the basic mathematically-geometric equations to initiate this endeavor. Another quote from [Riemannian_geometry] reads "Development of Riemannian geometry resulted in synthesis of diverse results concerning the geometry of surfaces and the behavior of geodesics on them, with techniques that can be applied to the study of differentiable manifolds of higher dimensions. It enabled the formulation of Einstein's general theory of relativity, made profound impact on group theory and representation theory, as well as analysis, and spurred the development of algebraic and differential topology."

The mathematical rendering of the spatial (3-dimensioal) "stress-energy-density" equations (energy-density geometric-equations describing the warped spatial region), as produced by applying a forcing-function or "geometry-to-physical-energy" translation (a coupling-constant in units of meters per Joule) to the "Riemannian-geometric-curvature equations" [a mathematical description of multi-dimensional (1,2,3,4….) geometric manifolds (also used in general relativity modeling as detailed above)], are applicable for 3-dimensional spatial static (no time-dependent motion or time-dependent characterization) systems. The Riemannian equations are applicable for time-dependent 4-dimensional systems as well as for 3- or 2-dimensional systems, and, in the present rendition with the "distinctively-generated metrics, [Supplementary Information equations (SI-4) through (SI-9)]", successfully produce an excellent mathematical-representation, ($1/r^4$), of Newtonian gravitational ($1/r^4$) and electromagnetic ($1/r^4$ and $1/r^6$) energy-density-formulated (think pressure) physical phenomena.

The present resulting geometric description of matter (mass-energy) successfully mimics the classical-physics electromagnetic(EM)- and gravitational-field models at

large radii of the distorted (warped) region, or energetic-matter region, but the "distorted-geometry-regional equations" (geometrical (force fields), a description of these same classical forces, depart significantly from Newtonian $1/r^4$ behavior at small radii (Newtonian form has an "undesirable" infinity at r = 0) and thereby produce a magnetic-field (spin) matter-mimic as well as a weak-field matter-mimic (beta decay and the Fermi-constant, a force-constant which describes the magnitude of the strength of the weak fields). A strong-field mimic is also mathematically-manifest without an infinity at the origin. There are no infinities or singularities, which are the undesirable hallmarks of pure classical Newtonian and electromagnetic physical models of matter, in this presently, geometrically-constructed, structural model [12].

Classically, physical forces have been characterized as independent entities, each with an associated strength (force constant), a Newtonian gravitational-force, an electromagnetic (electric and magnetic)-force, a weak-force describing the physical phenomenon of beta-decay and a strong-force describing short-range-repulsion effects. These forces are all manifested in the mathematical attributes (force-characteristics or force-constants) of this ONE distorted-geometric form (the solution to the Riemannian geometric tensor equations $\mathbf{Td_4^4}$, $\mathbf{Td_3^3}$, $\mathbf{Td_2^2}$ and $\mathbf{Td_1^1}$), that is, the "metric-solution" for a "material-like" distorted-space region of the manifold,

$$\mu'(r)= \frac{2}{(Iu + R0\, C1)\, \mathbf{f(r)}} \frac{u^2}{R0} = \frac{2\left(1 - u^3\right)}{(Iu + \gamma)} \frac{u^2}{R0} ,$$

where R0 is a "distortion-describing" radius and $Iu(u) = u\left[-1 + \frac{3}{4}u^3 - \frac{3}{7}u^6\right]$,

$$f(r) \stackrel{def}{=} \left(1 - u(r)^n\right)^{-1} \text{ with } u \equiv \frac{R0}{r} \quad and\ n = 3,$$

$$\text{and } R0 = \left(\alpha \frac{2}{3} \left(S \frac{ge}{2} \frac{Q}{3}\right)^2\right)^{1/3} \hbar c\, (mc^2)^{-1} ,$$

where S = spin quantity (S = 1 for the boson), ge = gyromagnetic ratio and Q/3 = an electric charge quantity (Q = 3 for an integral electronic-charge for the boson and Q = 1 and 2 if quarks are a physical entity (no other particles exhibit a fractional electronic-charge).

The geodesic equation for this spherically-symmetric geometry is

$$ds^2 = g_{11}\left[\, dr^2 + r^2 d\Omega \,\right] + g_{44} dt^2 = - e^{\mu}\left[\, dr^2 + r^2 d\Omega \,\right] + e^{\nu} dt^2 .$$

The "metric-solution" $\mu'(r)$ is further expressed fundamentally in the tensor energy-density (pressure) elements $\left[\mathbf{(Fd_{14})^2},\ \mathbf{\left(Fd_{mag}\right)^2}\ \mathbf{and}\ \mathbf{(Fd_{core})^2}\right]$.

The $\mu'(r)$ "metric solution" satisfies the "Minkowski" requirement that the "Metric-element coefficient-functions must be consistent with $\mathbf{T_{\mu\nu} = 0}$ (empty spacetime in the vicinity of a source mass) and must approach 1 as r approaches infinity, to become the Minkowski metric in spherical coordinates (the metric should be asymptotic flat.)"

This present "distorted geometry" "metric solution" satisfies the Minkowski requirement that

$$\mu'(r) = 0 \text{ at } r = \infty ,$$

and also intrinsically satisfies the condition that

$$\mu'(r) = 0 \text{ at } r = 0 ,$$

therefore no infinities (actually dictated by the tensor energy-densities; see SI).

This "particular form" of the metric quantity μ'(r) is the solution to the actual metric equation (SI-6d) generated by the "Maxwellian-like" and "material-like" tensor relationship expressed in equation (SI-4) taken from the SI:

$$\mu'' + \frac{f'}{f}\mu' + \frac{1}{2f}\left[(f - 3)(f - 1) + f \right] \mu'^2 + \frac{2\,\mu'}{r} = 0$$

with $\nu'(r) = [-2 + f(r)]\mu'(r)$ *and* $f(r) \stackrel{\text{def}}{\cong} (1 - u(r)^3)^{-1}$, (equation (SI-6a) from SI). It should be understood that this equation is the fundamental, constraining or restricting and defining differential equation for the metric entity $\mu'(r)$. The metric is not arbitrarily defined, it must satisfy this differential equation in the context of, and as dictated by, the Riemannian geometric equations (SI-4) and (SI-5); see the SI Supplementary Information for details. The inclusion of an "equation of state" as an additional "necessary or constraining" descriptor of "the distorted space," in conjunction with the four Riemann tensor energy-density equations, produces the metric-defining differential equation (SI-6d).

We use the "Black Hole Distortional Extremum [13] (a minimum hole mass) mass-energy" for calculating the "gravitational-interaction-strength" constant GG (see "Black Holes as Geometric Distortional Extrema [13]"). It should again be noted that the DG (distorted geometry) energy-density solution, equation (SI-7), is used universally to describe all "fundamental matter-forms, differing only in the magnitudes of the matter-descriptors, mass and charge, and the resulting coupling constant".

The geometrically-warped structure is constituted by a core-region within which the propagation-velocity, by virtue of the distorted-region metrics, is greater than c and exhibits a "partial light trapping phenomenon," facilitating and duplicating "classical black hole light-trapping" behavior. Warping or distorting the (our) spatial-manifold requires energy but with limits as to the degree of distortion (a "material-like" concept) thereby physically interpreting and describing fundamental-electromagnetic-particle structures (electrons, muons, tauons, and bosons) as well as gravitational (dark-matter(?), black-hole) structures. Given the geometrically described fundamental-particle structures, we then posit the dynamic character of the beta-

particle decay process with the production and physical characterization of the neutrinos.

2. **Theoretical Foundations**

Fundamental theoretical and mathematical foundations for this undertaking are presented as a Supplementary Information file.

3. **Calculational Methods**

3a. Calculational Methods for Geometric Modeling

For the "distorted-geometry, maximum-curvature" model, precise to the mass-characterization of the W-boson (see equations $\mathbf{Fd_{mag}}^{\mathbf{2}}$ and $\mathbf{Fd_{14}}^{\mathbf{2}}$ in the SI),

$$\mathbf{Fd_{mag}}^{\mathbf{2}}\, r^6\pi^4 = \frac{1}{8\pi\kappa} 2\ \mathrm{Rs}\ \mathrm{R0}^3\ \pi^4 = \frac{\pi^3}{2}\ m_w c^2\ \mathrm{R0_w}^3 =$$

$$\equiv \mathrm{GF(distorted\ geometry)}$$

or, $\mathrm{GF} = \left(\alpha\ \frac{2}{3}\right)^{1/3} \hbar c\ \frac{\pi^3}{2}\ \mathrm{R0_w}^2$ to accentuate the geometric dependence on "curvature,"

$$= \mathrm{GF(Fermi)} = 1.435851\ 10^{-62}\ \mathrm{Joule\ meters}^3 \quad . \qquad (1)$$

See SI for $\mathrm{Rs_W}$ and $\mathrm{R0_W}$, the radii for the $\mathrm{W^-}$boson.

Writing the classical energy-density for the $\mathrm{W^-}$boson as "the energy-density for the $\mathbf{Fd_{mag}}^{\mathbf{2}}$ (magnetic energy) component evaluated @ r = $\mathrm{R0_W}$", then

$$\rho_{boson} = \frac{U_{boson}}{\frac{4\pi}{3}\mathrm{R0_{boson}}^3} = 4.277\ 10^{46}\ \frac{\mathrm{Joule}}{\mathrm{meter}^3} \quad . \qquad (2)$$

The "distorted-geometry" mathematical symbols are

$$\kappa \equiv \kappa_{EM} + \kappa_G \cong \text{"}\mathrm{W_{boson}}\ \mathrm{coupling\ constant"} = \kappa_{EM} = \alpha\ \frac{\hbar c}{2}\left(\frac{Q}{3\ \mathrm{m_w} c^2}\right)^2 =$$

$$= 6.93\ 10^{-13}\ \frac{\mathrm{meters}}{\mathrm{Joule}} \quad (\mathrm{for}\ Q = 3), \quad (\mathrm{since}\ \kappa_G\ \mathrm{is\ infinitesimal\ compared\ to}\ \kappa_{EM}),$$

$$\alpha = \mathrm{fine\ structure\ constant}\,.$$

$$\hbar = \mathrm{Planck's constant}\,, \qquad c = \mathrm{velocity\ of\ light\ and}\ \mathrm{m_w} = \mathrm{boson\ mass}\,.$$

The distorted-geometry radial descriptor R0 (e.g., $\mathrm{R0_W}$ and $\mathrm{R0_\nu}$) is

$$R0 = \left(\alpha \frac{2}{3} \left(S \frac{ge}{2}\frac{Q}{3}\right)^2\right)^{1/3} \hbar c\ (mc^2)^{-1}\ , \qquad (2a)$$

where S = spin quantity (S = 1 for the boson), ge = gyromagnetic ratio and Q = a quantized (?) electric charge quantity (Q = 1,2 or 3 and Q = 3 for the boson), then,

$$R0_w = \left(\alpha \frac{2}{3}\right)^{1/3} \hbar c\ (m_w c^2)^{-1} \quad \text{and} \quad Rs_W = 2\,\kappa\, m_w c^2\ . \qquad (2b)$$

The energy-density structural nature of the 'distorted geometry" metric-solution [11] (Eq. (SI-7)),

$$\mu' = \frac{2(1-u^3)u^2}{(Iu - \gamma)R0}\ , \qquad u \equiv \frac{R0}{r}\ ,$$

gives rise inherently and comprehensively to the fundamental force quantities ($\mathbf{Fd_{12}}$, $\mathbf{Fd_{13}}$, and $\mathbf{Fd_{14}}$)**,** heretofore characterized as **independent** entities, that is, a weak-magnetic-force, an electric-force and a strong-force at the nuclear core. These force characterizations are here manifested as **r^{-6}, r^{-4}, and complex repulsive-core r^{-n} coupled-components** of the **"ONE geometric structure**". The structure is a balanced internal/external high-energy-density configuration, the difference in internal-pressure vs external-pressure manifested as particle mass-energy. The magnitude of the structural energy-density descriptor function is determined by the mass-energy or geometric-curvature with a geometry-to-energy coupling constant (meters/Joule) also dependent on these physical characteristics; the constant coupling-constant component G/c^4 (G = gravitational constant = 6.67408 $(10)^{-11}$ $m^3/(kg\ sec^2)$, of the composite EM-plus-Grav coupling-constant, describes gravitational structures when $\kappa_{EM} \cong \kappa_G$. The "distorted-geometry-solution (≡ DG)," is generated from Riemann's geometric description of a 4-dimensional spacetime manifold, applied at 3-dimensional localized warped- or distorted-space energy centers.

With the geometric success of mimicking the Fermi-constant as a particle-structure descriptor (the W^-boson), which is a " $mass_energy \times R0^3$ " product and which is a magnetic-energy-density weak-force maximum and a geometric-curvature maximum (inverse dependence on *r*) [11], we posit gravitational, electromagnetic and strong (core)-force "strength-of-interaction-DG" constants as energy-density coefficients of the various *r*-dependent components of a DG W-boson structure where as before,

$$GF(\text{Fermi costant}) = \frac{\pi^3}{2} R0_W{}^3\ m_w c^2 = \frac{\pi^3}{2}\left(\alpha \frac{2}{3}\right)\ (\hbar c)^3\ (m_w c^2)^{-2}\ .$$

However since such force ($(\mathbf{Fd_{14}})^2$, $\left(\mathbf{Fd_{mag}}\right)^2$ and $(\mathbf{Fd_{core}})^2$) entities are geometrically coupled entities, the classical "independently separable" model (**weak** plus e**lectromagnetic** (**EM**) plus **strong**) is not applicable. We instead use the energy-density maxima in the core region and in the extra-core region to establish the physical strengths (force constants) of the classical-differentiated force functions. To

elicit a present-day mental-model (picture or understanding) of the "force" concept, one would actually need to determine or describe the physical-spatial-region of interaction, a mathematical-integration process. We use the "Black-Hole Distortional Extremum [13] (a minimum hole mass) mass-energy" for calculating the "gravitational-interaction-strength" constant GG. Note that the "gravitational coupling-constant, $G*c^{-4} = 0.826\ 10^{-44}$ meters/Joule, is ~32-42 orders of magnitude smaller than the "EM (electromagnetic) coupling-constants" $8\pi\kappa_W$ or $8\pi\kappa_{electron}$.

3b. Calculational Method for the W-boson structure

The positive-pressure (positive energy-density) quantity, $(\mathbf{Fd_{14}})^{\mathbf{2}}$(electric, $\boldsymbol{Q \neq 0}$), for the W^-boson, evaluated at the core-radius functional-extremum, is

$$\mathbf{Fd_{14}}^{\mathbf{2}}(r^{-4}\ \text{component}) = \frac{Rs^2}{2}\frac{1}{8\pi\kappa}\frac{1}{r^4} = \frac{\hbar c}{8\pi}\,\alpha\frac{1}{R0_W{}^4} =$$

$$= \left(\frac{3}{2}\right)^{\frac{4}{3}}\left(\frac{1}{\alpha}\right)^{\frac{1}{3}}\frac{(m_w c^2)^4}{8\pi(\hbar c)^3}\ (\text{Joule/meter}^3)\,. \qquad (3)$$

3c. Calculational Method for the Neutrino structure

The "extremum" equation (1) can be rearranged to accentuate the geometric-structural elements giving rise to the "strength of interaction" quantity GF as follows;

$$\mathbf{Fd_{mag}}^{\mathbf{2}}\, r^6\pi^4 = \frac{1}{8\pi\kappa}2\ Rs\ R0^3\ \pi^4 = \frac{\pi^3}{2}\ m_w c^2\ R0_w{}^3 =$$

$$\equiv \text{GF(distorted geometry)} \qquad \text{or}$$

$$\frac{Q_w}{m_w c^2}\left(S_w\frac{ge_w}{2}\right)^{0.5} = \left(\frac{2\ GF}{\alpha\frac{2}{3}(\pi\hbar c)^3}\right)^{0,5}\,. \qquad (4)$$

We see then that the magnetic descriptors theoretically and mathematically (GF) allow for solutions describing extremum-structures other than the boson. Since the neutrino is dynamically fundamental to beta decay and the weak magnetic-field interaction, we ascribe to a neutrino structure, the extremum-magnetic characterization wherein, for example, using a neutrino mass [19-22] at 0.02 eV, an electric charge ($\neq 0$) is calculated at Q (neutrino) = $1.49(10)^{-12}$ (compared to Q = 3 for the electron or boson (see equation (2a)), or Q (neutrino) = $7.46(10)^{-11}$ for a neutrino

mass at 1.0 eV; such a charged neutrino structural description is presently not part of conventional "(Q = 0) modeling, which however is experimentally not verifiable".

3d. Calculational Method for energy-transition dynamics and the photon structure

In reference [15], we modelled "energy emitting structures" via a "black body construct" realized at the mass-level of a "fundamental particle" with a mass-energy = Universe-mass-energy. Here we posit such a "radiation-energy emitting" structure to describe photon emission dynamics, an energy-transition process. The "Planckian (Stefan-Boltzmann emitting body) power and energy distribution function" is integrated over the infinite energy spectrum and modelled as a spherical entity with radius R;

$$\mathrm{P(Planckian\ thermodynamics)} = \frac{\mathrm{dU}}{\mathrm{dt}} = -(\sigma \mathrm{T}^4)\ \mathrm{A(r)} \quad \text{and} \quad \mathrm{A(r)} = 4\pi\ \mathrm{R}^2. \qquad (5)$$

With

$$\mathrm{U} = \text{the distortional mass energy @ constant density(?)} = \rho\,,$$

$$\rho_{\mathrm{geo_boson}} \stackrel{\mathrm{def}}{=} \rho\mathrm{GF(DG)} = \frac{U_{boson}}{\frac{4\pi}{3}\ (R0_W)^3} = 4.277(10)^{46}\,\frac{\mathrm{J}}{\mathrm{m}^3}$$

and

$$\mathrm{Temp}_{\mathrm{geo_boson}} = \left(\rho\mathrm{GF}\ \frac{\mathrm{c}}{4\sigma}\right)^{1/4} = \ 2.74\ (10)^{15}\ K\,,$$

then

$$\frac{\mathrm{dU}}{\mathrm{dt}} = -\mathrm{c}(\pi\rho)^{\frac{1}{3}}\left(\frac{3U}{4}\right)^{\frac{2}{3}} \quad \text{and}$$

$$\mathrm{U(t)} = -\frac{1}{48}\ \pi\rho(\mathrm{c\ t})^3 + \mathrm{U0}\,. \qquad (6)$$

A final extinction time, wherein all of the structural energy has been depleted and converted to photon-energy, is reached at

$$\mathrm{tf} = \frac{4}{\mathrm{c}}\left(\frac{3\ \mathrm{U0}}{4\pi\rho}\right)^{1/3} = 4\,\frac{\mathrm{R}}{\mathrm{c}}\,, \qquad (7)$$

thereby producing a propagating directional photon (multi-particle production allowed) with a time-width tf and inherited blackbody and DG features; we assume a photon with velocity = c and exhibiting the "thermodynamic" body descriptors; "thermodynamic radiation" being understood as "EM radiation" at velocity c. The use of an "explosive" adjective to describe this dynamic feature is better appreciated when examining the enormous energy-densities (10^{48} Joules/ meter^3) or pressures (Pascals) within these "DG particle structures" (compare to a "stick of dynamite" at ~ 10^9 Pascals).

The extinction-time result can be interpreted as a "photonic-structural-descriptor" where $tf \equiv 1/\nu$ and $R \equiv \lambda/4$;

$$\lambda \nu = c\,; \tag{8}$$

the thermodynamic variable c has an electromagnetic "velocity of propagation" meaning. Electric charge features are not inherent to this development since "black bodies" have been modelled from thermodynamics and statistical mechanics theory although the charged boson-body characteristics in the form of the mass-energy density ρ, the total mass-energy U and the geometric radius feature R have been utilized; i.e. a simple conceptual model wherein "explosion-or energy-transition information" propagates physically throughout the exploding entity. The maximum-curvature DG-concept, from weak-force beta-decay modelling, produces a maximum energy limit at $R_{min} = R0_W$, a charge-induced, magnetic-field-$\left(\mathbf{Td_1^1} + \mathbf{Td_2^2}\right)$, r^{-6}, induced limit and therefore probably not the same limit as for $\left(\mathbf{Td_1^1}\right)$, r^{-4}, forces. In fact, the ratio of r^{-6} azimuthally-directed shear-type energy-densities to r^{-4} radially-directed energy-densities is

$$\frac{\mathbf{Fd_{mag}}^2}{\mathbf{Fd_{14}}^2} = \frac{8}{3}\left(\hbar c\, S\frac{Q}{3Mc^2}\right)^2\frac{1}{r^2} = \frac{8}{3}\left(\hbar c\,\frac{1}{mW\,c^2}\right)^2\frac{1}{r^2} = 372 \;\; @\, r = 0.5\, R0_W\,. \tag{9}$$

The "material properties" of the "distorted-space" are sufficiently significant in the azimuthal (shear) directions as to be responsible for the phenomenon of beta-decay, at least if the "mediator structure" is that of a W^-boson.

Examination of the DG-structure's energy-density profile in equations $\mathbf{Fd_{mag}}^2$ and $\mathbf{Fd_{14}}^2$ in the SI, and in Figure 1, reveals the marked departure from a structural "constant-energy-density" feature and the need to use a wavelength-dependent "spectral emissivity ελ". However the "low-emissivity" of these presently-modeled DG-structures can be understood as the reason for their stability, even though they exhibit short-lifetimes.

Although these distortional structures have been characterized at the outset as stable distortions, we have subsequently exploited the distortional form as the mediating

entities in distortional transition processes, suggesting that the structural stability can be of a transient nature and sensitive to environmental "fields."

3e. Calculational Method for the Gravitational structure (see Gravitational Results which incorporates the Methodology)

4. Results

4a. Boson structure results

The actual DG functional value at the energy-density maximum for the BOSON STRUCTURE is $7.64\ 10^{47}\ \frac{\text{Joules}}{\text{meter}^3}$ @ $r = 2.37\ 10^{-19}$ meters while the classical r^{-4} value is

$$\mathbf{Fd_{14}}^2(r^{-4}\text{ component}) = \frac{Rs^2}{2}\frac{1}{8\pi\kappa}\frac{1}{r^4} = 2.89\ 10^{45}\ \text{Joules/meter}^3\ @\ r = 2.37\ 10^{-19}\ \text{meters},$$

illustrating the magnitude of the contribution to $\mathbf{Fd_{14}}^2$ from the r^{-6} and other r^{-n} components.

Similarly, the negative-pressure (negative energy-density) core-maximum (for a W-boson structure) is

$$(\mathbf{Fd}_{boson\ core})^2(\text{max @ r} = 1.46\ 10^{-19}\text{meter}) = -2.51\ 10^{48}\ \text{Joule/meter}^3 \equiv$$

$$\equiv \text{STRONG FORCE energy-density maximum} = 869\ x\ \mathbf{Fd_{14}}^2(r^{-4}\text{ component}) =$$

$$= 87.5\ \text{x}\ \mathbf{Fd_{mag}}^2(r^{-6}\text{ component})\ . \qquad (10)$$

These field quantities are displayed in Fig.1.

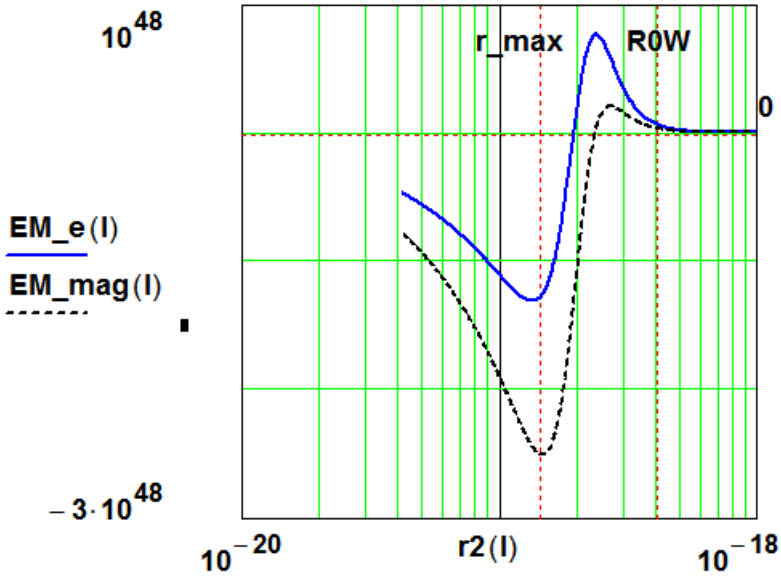

Fig.1. Distorted Geometry Electromagnetic Energy-Density (field) functions (**EM_e** for $\mathbf{Fd_{14}}^2$ and **EM_mag** for $\mathbf{Fd_{mag}}^2$**)** for the BOSONIC structure, illustrating the 'Strong-repulsive-force, Weak-force, and Electric-force" components. The ordinate in Joules/meter^3 is displayed in logarithmic form and the abscissa, the structural-radii r2, in meters in logarithmic values.

From the "energy-emission dynamics" model in ref. [15], and using the "corrected form" of equations 12-13 (ref.[15] corrected in ref.[16]), we can model and calculate the "lifetime" of this boson-structure as

$$\text{t(lifetime)} \stackrel{\text{def}}{=} \text{tf} = \frac{4}{c}\left(\frac{3\,U0}{4\pi\rho}\right)^{1/3} = \frac{4\,R_{boson}}{c} \stackrel{\text{def}}{=} \frac{4}{c}\,R0_W = 5.55\,10^{-27}\ \text{seconds}\,, \quad (11)$$

where U0 is the mass-energy of the "energy-emitting" body with a constant density ρ. The tf (lifetime) is approximately π/9 x Heisenberg time-interval , thereby satisfying the "transient-time requirement" for a "virtual -structure"; the W$^-$boson "Heisenberg-lifetime" is $\Delta t \geq \hbar/(2\,U_{boson}) = 4.1(10)^{-27}$ seconds. The transition lifetime for the "DG" W-boson provides an explicit form for energy-transitions of "distorted-geometry" structures and supports the "constant-density" "mathematical-structure" of equation (5).

Environmental fields [17,18] not included in the structural modeling would influence this "lifetime" as, for example, the stability behavior of a neutron "in" or "out" of the presence of nuclear fields. The present "energy emission" model is

elaborated in the following section for the "electromagnetic-radiation-emission structure."

4b. Neutrino structure results

A structural configuration describing a neutrino as a "stable energy-density mimic of the electron (e-neutrino)" is described as

$$\frac{\mathrm{Q_{e_neutrino}}}{(\mathrm{M_{e_neutrino}c^2})^2}\left(\mathrm{S}\frac{\mathrm{ge}}{2}\right)^{1/2} = \frac{\mathrm{Q_{electron}}}{(\mathrm{M_{electron}c^2})^2}\left(\frac{1}{2}\frac{2}{2}\right)^{1/2}. \qquad (12)$$

Such a "stable" structure would exhibit a charge Q at $1.53(10)^{-15}$ for a neutrino mass-energy of 0.02 eV or Q = $3.83(10)^{-12}$ for a neutrino mass-energy of 1.0 eV.

Assuming the electron-neutrinos are {1}, indeed "stable", muon or tauon or "reduced mass-form-electron", energy-density structures and {2}, the neutrino production process satisfies a "Fermi-constant weak magnetic-field interaction strength", we produce the electron-neutrino structural characteristics as

$$GF = A\left(\frac{Q_\nu}{U_\nu}\right)^2 \; and \;\; \rho_\nu = \rho_{elec}\;\left(or\;\rho_\mu\;\; or\;\rho_\tau\right);$$

$$with\; e.g.$$

$$U_{e_{neutrino}} = \left(GF\;\frac{2}{\pi^3}\;\boldsymbol{rde}\;\frac{4\pi}{3}\;\rho_{electron}\right)^{\frac{1}{2}}\; and$$

$$Q_{e_{neutrino}} = \left(\,U_{e_{neutrino}}\right)^2\left(\boldsymbol{rde}\;\rho_{electron}\;\frac{4\pi}{3}\;\alpha\frac{2}{3}\;(\hbar c)^3\right)^{-0.5};\quad \boldsymbol{rde} = \mathbf{0.0625}$$

$$(\boldsymbol{rde} = 1\;\; for\; muon\; decay)\,.$$

Then, $U_{e_neutrino} \equiv M_{\mathrm{e_neutrino}}\mathrm{c}^2 = 0.8118\; eV \;\; and \;\; \mathrm{Q_{e_neutrino}} = 1.01\;10^{-11}$. The energy-density of the **electron-neutrino** is $4.361\;(10)^{24}$ Joules/m^3.

The magnetic energy-density radius used in the decaying parent structure is the same as was used in the Fermi-constant calculation, namely, R0ν(electron neutrino) = $1.924(10)^{-15}$ meters and the electric energy-density radius is Rsν(electron neutrino) = $1.808\;(10)^{-31}$ meters (Eq. 2b) illustrating the vanishingly small electric-charge feature.

For a **muon-neutrino** structure,

$$U_{\mu_{neutrino}} \equiv M_{\mu_{\mathrm{neutrino}}}\mathrm{c}^2 = 1.388\;10^5\; eV \;\; and \;\; \mathrm{Q_{\mu_{neutrino}}} = 1.726\;10^{-6};$$

$$R0\nu_{\mu_neutrino} = 3.466\;10^{-17}\;m \;\; and \;\; \mathrm{Rs\nu_{\mu_neutrino}} = 3.092\;10^{-26} m\,;$$

$$Energy\ density\ of\ \mu_{\mu_neutrino} = 1.275\ 10^{35}\ \frac{Joules}{m^3}.$$

For a **tauon-neutrino** structure,

$$U_{\tau_{neutrino}} \equiv M_{\tau_{\text{neutrino}}} c^2 = 17.56\ MeV \quad and \quad Q_{\tau_{\text{neutrino}}} = 2.184\ 10^{-4};$$

$$R0\nu_{\tau_neutrino} = 6.905\ 10^{-18}\ m \quad and \quad Rs\nu_{\tau_\text{neutrino}} = 3.910\ 10^{-24}\ m\,.$$

$$Energy\ density\ of\ \mu_{\tau-neutrino} = 2.040\ 10^{39}\ \frac{Joules}{m^3}.$$

A "reduced-mass-coefficient," **rde**, has been introduced to approximately incorporate the "impact" of neighboring "third-body energy-density sources" on the local transitioning neutrino sources.

The reduced-mass-coefficient, **rde** = 1/16 for the e-neutrino, has been set to theoretically mimic the approximate (experimental mass upper limit < 0.8 eV for the e-neutrino) presently accepted and experimentally established e-neutrino-mass value (from tritium beta decay). The reduced-mass-coefficient, **rde** = 1/5 for the tau-neutrino to mimic the tau-neutrino-mass value from tauon decay (experimental mass upper limit < 18.2 MeV for the tau-neutrino). The reduced-mass-coefficient, **rde** = 1 (no "reduced mass" factor need be used) for structural calculations of the muon-neutrino thereby producing a neutrino-mass = 138.8 keV and mimics the "experimental muon-neutrino-mass value at < 150 keV" and should be considered the value for the un-adjusted (no **rde** modification) theoretical construct.

The neutrino mass-energy-densities, although exceptional, are within the DG geometric extrema limits(for the shear-stress components) expressed as ρ_boson ($4.277\ 10^{46}$ J/m^3) and ρ_electron ($6.977\ 10^{25}$ J/m^3).

4c. Photon structure results

A black-body emitted, propagating, DG photonic structure is simulated and mathematically detailed, as an example, for the Lyman-alpha line @ ƛ = 121.567 nanometers (labelled R0ν), in Fig.2; the simulation is also displayed in Fig.3 to better communicate the structure of the time-varying "energy-density fields".

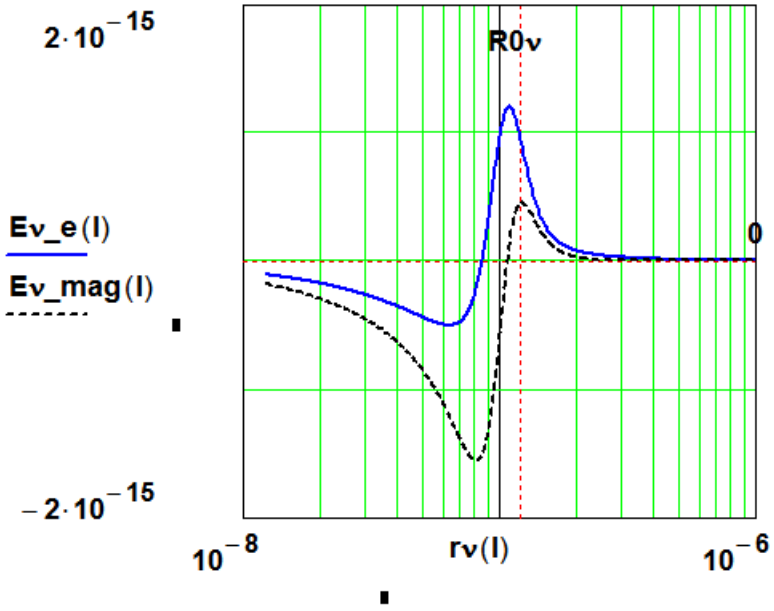

Fig.2. Distorted-Geometry-Photon Energy-Density (field) functions for the LYMAN-ALPHA_PHOTON (ƛ =121.567 10^{-9} meters = R0ν), illustrating the 'Strong-repulsive-force, Weak-force and Electric-force" components. The ordinate in Joules/meter^3 is displayed in logarithmic form and the abscissa, the structural radii rν, in meters in logarithmic values.

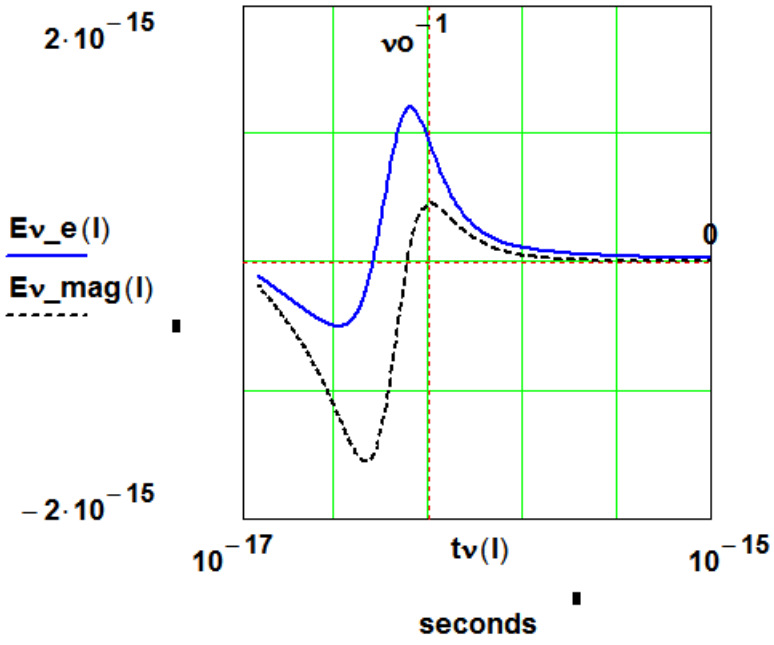

Fig.3. Distorted-Geometry-Photon Energy-Density (field) functions for the LYMAN-ALPHA_PHOTON illustrating the 'Strong-repulsive-force, Weak-force and Electric-force" components. The ordinate in Joules/meter^3 is displayed in logarithmic form and the abscissa in seconds in linear values. To emphasize the propagating energy, we have displayed the structural field character on a time scale. The actual time–extent of the photonic sphere (diameter at 2R) is double that shown in the core direction. Note the two physical-geometric facets of the Photon where Eν_e = 0 while Eν_mag = Eν_mag (max) and where Eν_e = Eν_e (max) while Eν_mag = 0, mimicking the behavior of an EM photon. The photon-frequency νo^{-1} condition occurs at the "extra-core" Eν_mag-maximum condition.

4d. Gravitational structure results

The positive-pressure (positive energy-density) quantity, $(\mathbf{Fd_{14}})^{\mathbf{2}}$**,** evaluated at the radius(r_max) of $(\mathbf{Fd_{14}})^{\mathbf{2}}$(max) , for the "HOLE_MIN" [13], gravitational structure, due to a maximum curvature, is

$$\mathbf{Fd_{14}}^{\mathbf{2}}(r^{-4}\text{ component}) = \frac{Rs^2}{2}\frac{1}{8\pi\,\kappa G}\frac{1}{r^4} = \frac{(\kappa G\ Mg)^2}{4\pi\,\kappa G}\frac{1}{r^4} =$$

$$= \frac{(\kappa G\ Mg\ c^2)^2}{4\pi\,\kappa G}\ \frac{1}{(3.69\ 10^{-3})^{\,4}} = 1.15\ 10^{47}\ (\text{Joule/meter}^3)$$

$$@\ r \equiv r_min = 3.69\ 10^{-3}\ \text{meters} \qquad (13)$$

and where $\kappa G = G\ c^{-4}$ and $Mg\ c^2 = \text{mass}_{\text{energy}}$ of HOLE MIN =

$$= 1.80\ 10^{41}\ \text{Joules.}$$

The actual DG functional value at the energy-density-maximum is $1.182\ 10^{48}\ \frac{\text{Joules}}{\text{meter}^3}$ @ r = r_min = $3.69\ 10^{-3}$ meters , again illustrating the magnitude of the contribution to $\mathbf{Fd_{14}}^{\mathbf{2}}$ from the r^{-6} and other r^{-n} components (see Fig.4).

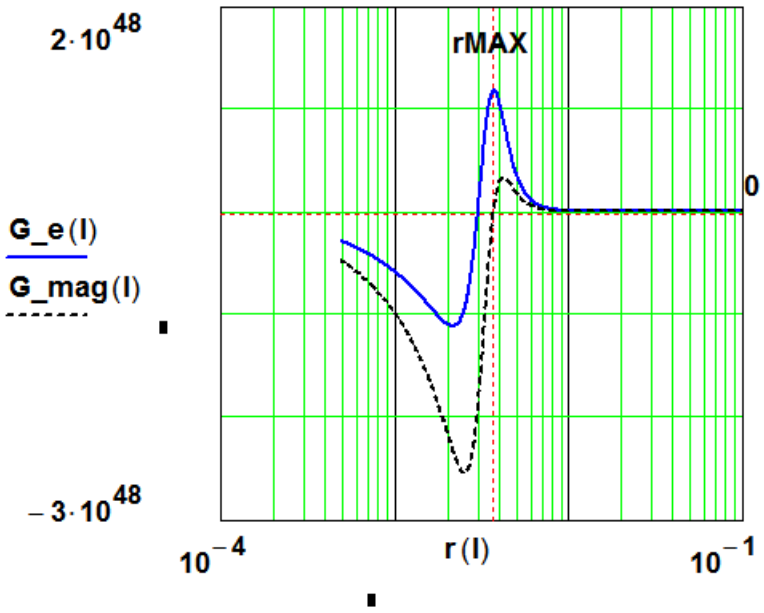

Fig.4. Distorted Geometry Gravitational Energy-Density (field) functions $\mathbf{G_e \equiv Fd_{14}}^{\mathbf{2}}$ and $\mathbf{G_mag \equiv Fd_{mag}}^{\mathbf{2}}$, (for the "HOLE-MIN" structure ≡ GRAVITATIONAL-force-field structure), illustrating a gravitationally-simulated 'Strong-grav.-force, Weak-grav.-force and grav. r^{-4}-force" components. The ordinate in Joules/meter^3 is displayed in logarithmic form and the abscissa, the structural radii r, in meters in logarithmic values.

The positive-pressure (positive energy-density) quantity, $(\mathbf{Fd_{14}})^{\mathbf{2}}$**,** evaluated at the radius of $(\mathbf{Fd_{14}})^{\mathbf{2}}(\max)$, for the "Milky Way Black-hole" [12] modeled as a gravitational structure is (but see reference [25], modeled as an EM structure)

$$\mathbf{Fd_{14}}^{\mathbf{2}}(r^{-4}\text{ component}) = \frac{Rs^2}{2}\frac{1}{8\pi\,\kappa G}\frac{1}{r^4} = \frac{(\kappa G\ Mg\ c^2)^2}{4\pi\,\kappa G}\frac{1}{r^4} =$$

$$= \frac{(\kappa G\ Mg\ c^2)^2}{4\pi\,\kappa G}\ \frac{1}{(1.34\ 10^{10})^4} =$$

$$= 8.70\ 10^{21}\ (\text{Joule/meter}^3)\ @\ r = 1.34\ 10^{10}\ \text{meters} \qquad (14)$$

and where $\kappa G = G\ c^{-4}$ and $Mg =$ mass of Black Hole Sagittarius $A* =$

$$= 4.154\ 10^6\ \text{solar masses.}$$

The actual DG functional value at the $\mathbf{Fd_{14}}^{\mathbf{2}}$ energy-density maximum is $8.95\ 10^{22}\ \frac{\text{Joules}}{\text{meter}^3}$ @ $r = 1.34\ 10^{10}$ meters , again illustrating the magnitude of the contribution to $\mathbf{Fd_{14}}^{\mathbf{2}}$ from the r^{-6} and other r^{-n} components (see Fig.5).

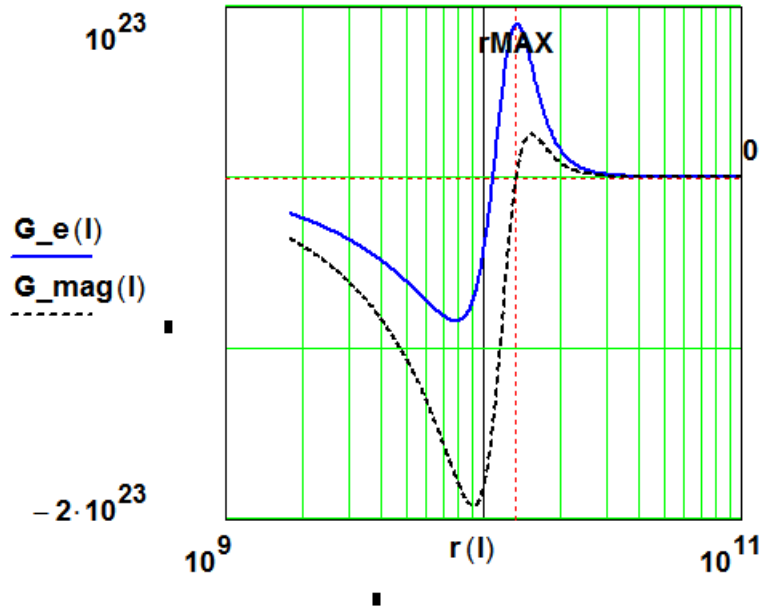

Fig.5. Distorted Geometry Gravitational Energy-Density (field) functions $\mathbf{G_e \equiv Fd_{14}}^{\mathbf{2}}$ and $\mathbf{G_mag \equiv Fd_{mag}}^{\mathbf{2}}$, for the MILKY WAY Black-Hole, illustrating a gravitationally-simulated 'Strong-grav.-force, Weak-grav.-force and grav. r^{-4}-force" components. The ordinate in Joules/meter^3 is displayed in logarithmic form and the abscissa, structural radii r, in meters in logarithmic values. rMAX is the value of the radius at the Black-Hole maximum of $\mathbf{G_e \equiv Fd_{14}}^{\mathbf{2}}$.

Similarly, the negative-pressure (negative energy-density) core-maximum (for this black-hole gravitational structure) is

$$\left(\mathbf{Fd}_{\boldsymbol{grav\ core}}\right)^{\mathbf{2}}(\text{max @ r} = 9.08\ 10^{9}\text{meter}) = -1.92\ 10^{23}\ \text{Joule/meter}^3. \qquad (15)$$

Finally, a "gravitational representation" of the Fermi-constant, a measure of a maximum-curvature minimum-radius structure, can be calculated according to the Fermi definition (a magnetic field descriptor) as

$$\text{Gravitational interaction strength constant} \equiv GG \equiv \frac{\pi^3}{2}\, m_G c^2\, R0_G{}^3 =$$

$$= \frac{\pi^3}{2}\, m_G c^2\, (\gamma\, 2\, G\, c^{-4} m_G\, c^2)^3 \text{ with } \gamma = \frac{3.275}{2} \text{ and}$$

$$m_G c^2 = \text{Black hole mass energy minimum [12]},$$

where

$$G\, c^{-4} = \text{gravitational coupling constant} = 8.26\, 10^{-45}\, \frac{\text{meters}}{\text{Joule}}\ ,$$

then $$GG = 3.2242\, 10^{35}\ \text{Joule meter}^3\ . \qquad (16)$$

5. Discussion

It has been shown in the present work that the distorted-space, or distorted geometry (DG), model of matter, as applied to fundamental-particle constructs (boson, tauon, muon, electron, neutrino, neutron, and a gravitational-hole), can produce structures satisfying "particle mass-energy-transition" or "force-field" dynamics. Earlier successful mimicking [12] of "Fermi-described beta decay" has been extended to a force-field description of "classical radiation-emission", neutrinos, and a "gravitational energy-transition" entity as well. The DG (distorted geometry)-neutrino mass-energies and electric-charges have been theoretically calculated in good agreement with experimental measurements.

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Availability of Data and Materials: All data generated or analyzed during this study are included in this published article [and its supplementary information files]; Contact is corresponding author **Dale R. Koehler, Ph.D**.

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SI Supplementary Information

Supplementary Equations

The Neutrinos and Force Constants of Geometric Force-Field structures wherein a *First-Principle Distorted-Geometry* mimics Mass(*hydrostatic stress*) and Electric Charge(*shear stress*)

Dale R. Koehler

For the presently described spherically symmetric Riemannian-geometric tensors, the Maxwellian electromagnetic tensor and the associated field tensor $\mathbf{F_{1\mu}}$ can be constructed according to equation (SI-1), where the only surviving field tensor components are (following the symbolism and development of Tolman [SI-1]):

$$ds^2 = g_{11}\left[\, dr^2 + r^2 d\Omega \,\right] + g_{44}dt^2 = -\, e^{\mu}\left[\, dr^2 + r^2 d\Omega \,\right] + e^{\nu}dt^2 \,,$$

$$\mathbf{F_{21} = -\, F_{12}} \,, \quad \mathbf{F_{13} = -\, F_{31}} \quad \text{and} \quad \mathbf{F_{14} = -\, F_{41}} \,, \quad \text{i.e.}$$

$$\mathbf{T^{\mu\nu} = -\, g^{\nu\beta}F^{\mu\alpha}F_{\beta\alpha} + \frac{1}{4}\, g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}} \quad \text{or} \quad \mathbf{T^{\mu\mu} = -\, g^{\mu\mu}F^{\mu\alpha}F_{\mu\alpha} + \frac{1}{4}\, g^{\mu\mu}F^{\alpha\beta}F_{\alpha\beta}} \,,$$

then

$$\mathbf{T_4^4 = \frac{\left(F_{12}F^{12} + F_{13}F^{13} - F_{14}F^{14}\right)}{2}}, \quad \mathbf{T_1^1 = \frac{\left(-\, F_{12}F^{12} - F_{13}F^{13} - F_{14}F^{14}\right)}{2}},$$

$$\mathbf{T_2^2 = \frac{\left(-\, F_{12}F^{12} + F_{13}F^{13} + F_{14}F^{14}\right)}{2}} \quad \text{and} \quad \mathbf{T_3^3 = \frac{\left(F_{12}F^{12} - F_{13}F^{13} + F_{14}F^{14}\right)}{2}} \,; \qquad \text{(SI-1)}$$

$$\mathbf{T_4^4 = -\left(T_1^1 + T_2^2 + T_3^3\right)} \,. \qquad \text{(SI-2)}$$

The resultant field quantities are

$$\mathbf{(F_{14})^2 = -\left(T_4^4 + T_1^1\right)g_{11}\, g_{44} = \left(T_2^2 + T_3^3\right)g_{11}\, g_{44}} \,,$$

$$\mathbf{(F_{12})^2 = -\left(T_2^2 + T_1^1\right)g_{11}\, g_{11}} \quad \text{and} \quad \mathbf{(F_{13})^2 = -\left(T_3^3 + T_1^1\right)g_{11}\, g_{11}} \,. \qquad \text{(SI-3)}$$

Therefore, we see that the static-spherically-symmetric Maxwellian tensors, (SI-1) and (SI-2), exhibit the same stress and energy relationship as the "equation-of-state geometric-tensors [SI-4], The present geometric-modeling endeavor, with its Maxwellian-tensor-form mimicking-component, has produced the fundamental and limiting agent for the currently-studied distorted geometry, namely a particular constraining functional relationship between the geometry-defining tensors (for an empty-space geometry, all of the components of the energy-momentum tensor are zero). In characterizing this simple equation (SI-4) as an "equation-of-state" and as a restricting distortional-model tensor relationship, we thereby elicit the metric-defining differential equations for such a family of geometric distortions.

We constrain the modeling therefore by requiring that the descriptive stress-energy tensors satisfy this "constitutive relation" or "equation-of-state" between the temporal and spatial tensor-curvature elements, namely we require

$$\mathbf{Td}_{\mathbf{4}}^{\mathbf{4}} = -\left(\mathbf{Td}_{\mathbf{1}}^{\mathbf{1}} + \mathbf{Td}_{\mathbf{2}}^{\mathbf{2}} + \mathbf{Td}_{\mathbf{3}}^{\mathbf{3}}\right). \qquad \text{(SI-4)}$$

We have introduced the explicit distortional-tensor symbolism **Td** for the geometric quantities. Contrast this perspective with cosmological renditions of geometric curvature structure resulting from "matter" causation, wherein several "equations of state" relating to the "matter" variables ρ (density) and p (pressure) have been forthcoming [SI-3] where $p = \sigma \rho$ and where σ varies from -1 to +1.

Since, inherently, in the geometric "equation-of-state" constraint, the requirement that the descriptive stress-energy tensor, **Td**, be Maxwellian in nature, requires that the mimicking process be limited to asymptotically flat-space regions of the manifold since $1/r^2$ field behavior does not adequately describe elementary-particle structural-detail.

The calculational treatment employs the isotropic coordinate description of equation (SI-1) and utilized by Tolman [SI-1], where the system of equations represented by equation (SI-1), is shown more explicitly in equation (SI-5) in mixed tensor form;

$$8\pi\kappa T_1^1 = -e^{-\mu}\left[\frac{\mu'^2}{4} + \frac{\mu'\nu'}{2} + \frac{\mu' + \nu'}{r}\right] + e^{-\nu}\left[\ddot{\mu} + \frac{3}{4}\dot{\mu}^2 - \frac{\dot{\mu}\dot{\nu}}{2}\right],$$

$$8\pi\kappa T_2^2 = -e^{-\mu}\left[\frac{\mu''}{2} + \frac{\nu''}{2} + \frac{\nu'^2}{4} + \frac{\mu'+\nu'}{2r}\right] + e^{-\nu}\left[\ddot{\mu} + \frac{3}{4}\dot{\mu}^2 - \frac{\dot{\mu}\dot{\nu}}{2}\right] = 8\pi\kappa T_3^3,$$

$$8\pi\kappa T_4^4 = -e^{-\mu}\left[\mu'' + \frac{\mu'^2}{4} + \frac{2\mu'}{r}\right] + e^{-\nu}\left[\frac{3}{4}\dot{\mu}^2\right],$$

$$8\pi\kappa T_4^1 = +e^{-\mu}\left[\dot{\mu}' - \frac{\dot{\mu}\nu'}{2}\right],$$

$$8\pi\kappa T_1^4 = -e^{-\nu}\left[\dot{\mu}' - \frac{\dot{\mu}\nu'}{2}\right]. \qquad \text{(SI-5)}$$

Metric coupling, that is terms such as μ'ν,' are apparent in the fundamental curvature equations. The usual notation, where primes denote differentiation with respect to the radial coordinate r and dots denote differentiation with respect to the time coordinate t, is employed. We are considering the static case (where total differentiation replaces partial differentiation) as was also used for Schwarzschild's (gravitational) interior and exterior solutions for the model of an incompressible perfect-fluid sphere of constant density surrounded by empty space [SI-1]. In that work a zero-pressure surface-condition and matching and normalization of the interior and exterior metrics at the sphere radius were used as boundary conditions.

Tolman [SI-1] has shown that the energy of a "quasi-static isolated system" can be expressed as "an integral extending only over the occupied space", which we will allow to extend to infinity, and where the total energy of such a sphere is therefore expressed as

$$U(\text{sphere total}) = \int_0^{\infty}\left(T_4^4 - T_1^1 - T_2^2 - T_3^3\right)\sqrt{\left(-g_{11}\right)^3 g_{44}}\; 4\pi\, r^2\, dr = M_{\text{sphere}}\, c^2 \quad . \qquad \text{(SI-6)}$$

This mass-energy representation will be used throughout in calculating the distortional mass-energies. The distortional-tensor energy-density amplitudes manifested in these presently calculated geometric representations are both negative and positive, that is, there are both negative energy-density [SI-2] and positive energy-density regions internal to the distortions. Since geometric distortional fields arise from the same energy-density tensors, the negative energy-density geometric regions are also sources of negative energy-density field quantities.

In the current procedural characterization, one has sufficient information to move to a solution of the differential equations without explicitly stating any "material" energy densities, thereby maintaining the spatial-distortion causative-perspective. The metric μ and ν solutions can consequently determine the resultant energy tensors. The consequent equation however still involves both temporal, ν, and spatial, μ, variables and accordingly requires one further qualifying distortional relationship to afford solution. In generalized format we write such a – metric-relationship as

$$\nu' = [-2 + f(r)]\mu' \text{ (conveniently simple)} \qquad \text{(SI_6a)}$$

with f(r) further defined in order to mimic "Maxwellian" and "Schwarzschildian" behavior; "Schwarzschildian" behavior defined as "Einstein equations with r and t constant so that $dr^2 = 0$ and $dt^2 = 0$ and then the metric becomes $ds^2 = r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$ which is the line element for the surface of a sphere meaning the metric is spherically symmetric. Metric-element coefficient-functions must be consistent with $\mathbf{T_{\mu\nu} = 0}$ (empty spacetime in the vicinity of a source mass) and must approach 1 as r approaches infinity, to become the Minkowski metric in spherical coordinates (the metric should be asymptotic flat)."

The "Kerr" metric is an exact solution of the Einstein equations, generalizing the Schwarzschild metric to represent a "spinning black hole."

Examination of the Maxwellian field quantities $\mathbf{F_{12}}$ and $\mathbf{F_{14}}$ helps in regard to defining f(r) and for the present modeling work, we use the fundamental distortion-defining form

$$f(r) \stackrel{\text{def}}{=} \left(1 - u(r)^n\right)^{-1} \text{ with } u(r) \stackrel{\text{def}}{=} \left({}^{R0}/_{r}\right). \qquad \text{(SI-6b)}$$

R0 is a distortional-characteristic, or normalizing, radius. The present functional construction can be compared with the Schwarzschild solution [SI-1] in isotropic coordinates where

$$g_{11} \stackrel{\text{def}}{=} -\exp(\mu) = -\left(1 + u(r)\right)^4 \text{ and } g_{44} \stackrel{\text{def}}{=} \exp(\nu) = \left[\frac{1 - u(r)}{1 + u(r)}\right]^2 \text{ or}$$

$$\nu' = -\left(1 - u(r)\right)^{-1}\mu' \text{ with } u(r) \stackrel{\text{def}}{=} \frac{Rs}{4r}; \quad Rs \stackrel{\text{def}}{=} \text{Schwarzschild radius}\,.$$

It will be found in the following development that the simplest (n = 3) distortional form is required to produce zero energy-densities at r = 0, the radial origin of the geometric structure,

and a magnetic energy-density mimic at large radii, as well as producing the "metric Schwarzschildian" behavior and the geometric-physical characteristics of the modeled particles.

Equations (SI-5) can be written in a static system as

$$8\pi\kappa\, \mathbf{Td_1^1} = -e^{-\mu}\mu' \left[\frac{\mu'}{4}(2f - 3) + \frac{f - 1}{r} \right],$$

$$8\pi\kappa\, \mathbf{Td_2^2} = -e^{-\mu} \frac{\mu'}{2}\left[\frac{1}{f}f' + \frac{1}{2f}(3 - 2f)\mu' + \frac{1}{r}(1 - f)\right] = 8\pi\kappa\, \mathbf{Td_3^3},$$

$$8\pi\kappa\, \mathbf{Td_4^4} = e^{-\mu}\mu' \left[\frac{1}{f}f' + \frac{\mu'}{4f}(2f - 3)(f - 2)\right]. \qquad \text{(SI-6c)}$$

Imposing the "equation of state" relationship (SI-4), in the tensor eq. (SI-6c) for $\mathbf{Td_4^4}$, the "metric differential equation" (SI-6d) is created .

$$\mu'' + \frac{f'}{f}\mu' + \frac{1}{2f}\left[(f - 3)(f - 1) + f \right]\mu'^2 + \frac{2\mu'}{r} = 0 . \qquad \text{(SI-6d)}$$

A solution to this equation (SI-6d) is found to be

$$\mu'(r, f(r)) = \frac{2}{r^2 f(r)}\, H(r) \text{ with } H(r)^{-1} = \int \frac{(f - 3)(f - 1) + f}{(r f)^2}\, dr + C1 \overset{\text{def}}{\equiv} \frac{Iu}{R0} + C1 \quad \text{or}$$

$$\text{for } f \equiv f((\text{SI-6b})), \text{ and } n=3, \qquad \mu'(r) = \frac{2(1 - u^3)}{(Iu + R0\, C1)}\frac{u^2}{R0} =$$

$$= \frac{2(1 - u^3)}{(Iu - \gamma)}\frac{u^2}{R0} \quad \left(\begin{array}{c} \text{for} \\ \gamma \overset{\text{def}}{\equiv} -C1\, R0 \end{array} \right) \qquad \text{(SI-7)}$$

where

$$Iu(u) = u\left[-1 + \frac{3}{4}u^3 - \frac{3}{7}u^6\right] \text{ and}$$

$$\mu'(r) = 0 \text{ at } r = \infty \text{ and } r = 0 .$$

"For the special case where f = 1", equation (SI-6d) becomes

$$\frac{d\mu'}{\mu'} + \frac{1}{2}d\mu + \frac{2}{r}dr = 0 \quad \text{with solution} \quad \mu' = \sqrt{B}\, e^{-\frac{\mu}{2}} r^{-2} .$$

$$\text{Then} \quad e^{\mu} = \frac{B}{4r^2} + C \quad \text{and} \quad \nu' = -\mu' ;$$

$$8\pi\kappa\, \mathbf{Td_4^4} = e^{-\mu}\mu' \left[\frac{\mu'}{4}\right] = \left[\frac{B}{4}\right] r^{-4} ,$$

$$8\pi\kappa\, \mathbf{Td_1^1} = e^{-\mu}\mu' \left[\frac{\mu'}{4}\right] \quad \text{and}$$

$$8\pi\kappa\, \mathbf{Td_2^2} = -e^{-\mu}\,\frac{\mu'}{4}\left[\mu'\right] \;=\; 8\pi\kappa\, \mathbf{Td_3^3}\,.$$

This "special case" is a description of a "Maxwellian Perfect-Fluid-like Distortion" with $\mathbf{p0}$ (pressure) $= \boldsymbol{\rho 0}$ (mass-energy density)**;** the definition of the **"**Maxwellian Perfect-Fluid**"** being $\mathbf{-(Td_1^1 + Td_2^2 + Td_3^3) = p0}$ and $\mathbf{Td_4^4 = \rho 0}$. In this case, however, the metric-variables μ', ν' and the energy-density tensor quantities $\mathbf{Td_4^4}$, $\mathbf{Td_3^3}$, $\mathbf{Td_2^2}$ and $\mathbf{Td_1^1}$ **,** are all infinite at r = 0.

The μ' metric solution satisfies the "Minkowski" requirement that the "Metric-element coefficient-functions must be consistent with $\mathbf{T_{\mu\nu} = 0}$ (empty spacetime in the vicinity of a source mass) and must approach 1 as r approaches infinity, to become the Minkowski metric in spherical coordinates (the metric should be asymptotic flat)." Moreover the μ' metric solution, μ'(r) also satisfies the heretofore unsatisfied requirement that μ'(r = 0) = 0 for no infinities.

The metric quantities g_{44} and g_{11} behave consistently with the classical notion of a "Lorentzian-Riemannian smooth manifold with a continuous two-index metric tensor field, non-degenerate at each point of the manifold, which is a basic ingredient of general relativity and other metric theories of gravity"[SI-1].

Both mass and field equations are dominated by the μ*'* function, but the radial region near the geometric origin requires a precise functional description of the metric quantities g_{44} and g_{11} to accurately characterize the fundamental physical quantities. In fact, as will be shown, the mass-energy quantity is almost exclusively generated by the transition region from "flat-space" to "distorted-space."

Written in the "u" variable form,

$$8\pi\kappa\, \mathbf{Td_1^1} \;=\; -e^{-\mu}\frac{1}{(Iu-\gamma)}\left(\frac{u^2}{R0}\right)^2\left[\,2\,u^2+(3\,u^3-1)\,\frac{1-u^3}{(Iu-\gamma)}\,\right],$$

$$8\pi\kappa\, \mathbf{Td_2^2} = \;\; e^{-\mu}\frac{1}{(Iu-\gamma)}\left(\frac{u^2}{R0}\right)^2\left[\,4\,u^2+(3u^3\,-1)\,\frac{(1-u^3)^2}{(Iu-\gamma)}\right],$$

$$8\pi\kappa\, \mathbf{Td_4^4} = -\,8\pi\kappa\,(\,\mathbf{Td_1^1} + \mathbf{2}\,\mathbf{Td_2^2}\,) \;\text{ since }\quad \mathbf{Td_3^3 = Td_2^2}$$

or

$$\mathbf{Td_4^4} \;=\; e^{-\mu}\frac{1}{8\pi\kappa(Iu-\gamma)}\left(\frac{u^2}{R0}\right)^2\left[-6\,u^2-(3u^3\,-1)\,\frac{(2u^3-1)(u^3-1)}{(Iu-\gamma)}\right] \qquad \text{(SI-8)}$$

and $\quad 8\pi\kappa\,(\,\mathbf{Td_2^2} + \mathbf{Td_1^1}\,) = \; e^{-\mu}\frac{1}{(Iu-\gamma)}\left(\frac{u^2}{R0}\right)^2\left[\,2\,u^2-(3u^3\,-1)\,\frac{(1-u^3)u^3}{(Iu-\gamma)}\right]$

leading to

$$(\mathbf{Fd_{14}})^2 = -\,g_{11}g_{44}\left(\mathbf{Td_4^4} + \;\mathbf{Td_1^1}\right) = g_{11}g_{44}\left(2\;\mathbf{Td_2^2}\,\right) \qquad \text{and}$$

$$(\mathbf{Fd_{14}})^2(r\to\infty) \overset{\text{def}}{\cong} \left(\frac{Rs}{2}\right)^2\frac{2}{8\pi\kappa}\,\frac{1}{r^4} = \frac{Rs^2}{2}\frac{1}{8\pi\kappa}\,\frac{1}{r^4} \overset{\text{def}}{\cong} \left(\frac{q}{4\pi\varepsilon o\, r^2}\right)^2\frac{\varepsilon o}{2}.$$

$$(\mathbf{Fd_{12}})^2 + (\mathbf{Fd_{13}})^2 = 2\, g_{11} g_{11} \left(\frac{\mathbf{Td_4^4 - Td_1^1}}{2} \right) \overset{\text{def}}{\cong} \mathbf{Fd_{mag}}^2 =$$

$$= -\,2\, g_{11} g_{11} \left(\mathbf{Td_1^1} + \mathbf{Td_2^2} \right) \quad \text{and}$$

$$(\mathbf{Fd_{12}})^2 + (\mathbf{Fd_{13}})^2 (\mathbf{r} \to \infty) = 2\, Rs\, R0^3 \frac{1}{8\pi\kappa} \frac{1}{r^6} \overset{\text{def}}{\cong}$$

$$\overset{\text{def}}{\cong} \frac{\mu o}{2} \left(\frac{\mu_{spin}}{2\pi} \right)^2 \frac{1}{r^6}$$

where $\mu_spin \overset{\text{def}}{\cong} \left(\frac{g_e}{2} \frac{Q\, e}{3M} \right) S\, \hbar$ and $g_e = 2.0023193043\ 6$ (for the electron) .

To illustrate the dependence of the tensor quantities on the metric quantity "near the radial origin," we can show that for "non-infinite" behavior, n must be ≥ 3 ; for example,

$8\pi\kappa \mathrm{Td}_1^1(r \to 0) = e^{-\mu} \left[\frac{2}{3\, R0^2} (2n+1) u^{2-n} \right] = 0$ if n ≥ 3 and $e^{-\mu} = e^{\frac{-2(2n+1)}{3} \frac{}{n} u^{-n}} = e^{-0}$.

After solving for μ and subsequently forming the asymptotic metric form, that is, the expanded large-radius metric, we have equated the distortional-geometric form to a Schwarzschild form with the result that 2/C1 = -Rs. See for comparison the Reissner [SI-4] and Nordstrom [SI-5] metric form constructed in the standard coordinate system for an object with mass "m" and charge "e".

Having constrained the first order large-r (or r→∞) metric behavior to the Schwarzschild value $Rs/r \overset{\text{def}}{\cong} 2\kappa \frac{Mc^2}{r} = -2/(C1\ r)$, we have ascribed to the model the geometrically-generated characteristic of "mass-energy" via an undetermined "microscopic geometrostatic coupling-factor", *κ*. In this characterization, C1 must also be negative to satisfy the "pseudo-magnetostatic" field constraint elaborated below. Incorporation of the $\mathbf{Fd_{14}}^2 \overset{\text{def}}{\cong} \left(\frac{q}{4\pi\varepsilon o\, r^2} \right)^2 \frac{\varepsilon o}{2}$ term (q is the charge quantity and εo is the permittivity constant) implies an inclusion of the geometrically-generated characteristic of "electrostatic charge-energy" in the model (also see Tolman [SI-1]).

Because of the fundamental geometrostatic equation-of-state relationship, the $\mathbf{Td_2^2}$ and $\mathbf{Td_3^3}$ tensor elements are considered constructs of the basis tensors $\mathbf{Td_4^4}$ and $\mathbf{Td_1^1}$, since $\mathbf{Td_2^2 = Td_3^3 = -\left(Td_4^4 + Td_1^1\right)/2}$, and therefore not basis elements themselves. Also for the geometrostatic case, the $\mathbf{Fd_{12}}^2$ and $\mathbf{Fd_{13}}^2$ field quantities are identical and therefore considered functionally addable, at the tensor expression level, to form the single field quantity $\mathbf{Fd_{mag}}^2$. Although there are no "moving" charges in this static spherically symmetric model, the geometric field tensor entity $\left(\mathbf{Fd_{mag}}\right)^2$ is non-zero and therefore both "pseudo-electrostatic" and "pseudo-magnetostatic" distortional-fields are produced. The "pseudo-magnetostatic" distortional-field source-strength is that of a "magnetic-dipole of magnitude to produce the dipole-axial-field."

The electrostatic and magnetostatic constraints (where h = Planck's constant q = particle electric-charge, M = particle mass, μo = permeability constant and c = speed of light ≡ the flat-space propagation-velocity within the geometric manifold), constructed at large radii with r^{-4} and r^{-6} behaviors, have therefore produced (or required for mimicking), in conjunction with the "metric mass-energy" constraint, that the coupling constant be a variable, $\kappa o \stackrel{def}{\equiv} \kappa = \mu o \left(\frac{\mu_{spin}}{\hbar\, c\, S\, g_e}\right)^2 /2\pi$. Having mimicked both mass-energy and electromagnetic-energy behavior, the coupling constant is a function of both source entities, the electric charge, and the mass, but in a combined fashion expressed through the single physical descriptor "μ_spin." These constraints have also determined the geometrostatic quantities, R0 (the distortional-characteristic radius), and C1 (the metric-derivative integration constant). The results are summarized in equation (SI-9); κ, C1 and R0 have been expressed to show more explicitly the charge q and mass M dependence. The three geometric descriptors have been fixed by the two physical descriptors since C1 = C1(κ,M*)* and R0 = R0(κ,M*)*.

$$\kappa o = \frac{\alpha\, \hbar c \left(\frac{Q}{3}\right)^2}{2\left(Mc^2\right)^2} = \frac{\mu o}{2\pi}\left(\frac{\mu_spin}{\hbar c\, S\, g_e}\right)^2 \quad , \quad C1 = -\frac{2}{Rs} = -\frac{1}{(\kappa o\, Mc^2)} \qquad \text{(SI-9)}$$

$$\text{and} \quad R0^3 = \frac{1}{3}\left(\hbar c\, S\, g_e\right)^2 \frac{\kappa o}{Mc^2} = \frac{\mu o}{4\pi}\frac{\mu_spin^2}{Mc^2} \quad .$$

A structural-constant is manifested in the 1st mass-moment, R0 Mc^2, or

$$R0\, Mc^2 = \beta\, \hbar c \qquad \text{with} \qquad \beta = \left[\left(S\frac{ge}{2}\frac{Q}{3}\right)^2 \alpha\, \frac{2}{3}\right]^{1/3} .$$

Although a classical presentation would not require a "quantization" restriction, we have included this feature to reflect the theoretically used quark concept.

Finally, the geometrostatic sphere "mass-energy" equation must be satisfied for internal integrity and modeling success.

By using the "equation-of-state" relationship for the geometric tensors, the mass equation integrand becomes $2\, \mathbf{Td_4^4} \sqrt{\left(-g_{11}\right)^3 g_{44}}\, 4\pi r^2$, a function of the single geometric tensor element $\mathbf{Td_4^4}$. Writing the tensor component explicitly, we have for the "sphere mass energy," a product of tensor energy-density and the distortional differential volume element.

$$U(\text{sphere}) = \int_0^\infty 2\, Td_4^4 \sqrt{(-g_{11})^3 g_{44}}\, 4\pi\, r^2\, dr \stackrel{def}{\equiv} M_{sphere}\, c^2$$

$$\text{or} \qquad U(\text{sphere}) = \int_0^\infty \rho Mass\, dr \qquad \text{where}$$

$$\rho Mass = \frac{1}{Iu - \gamma}\left[\frac{f^{-1}}{Iu - \gamma}\left(2 - \frac{3}{f}\right)\left(1 - \frac{2}{f}\right) - 6u^2\right]\frac{u^2}{\kappa}\left(|g_{11}|g_{44}\right)^{0.5} \quad \text{and}$$

$$\gamma \stackrel{\text{def}}{=} -C1\ R0 = \frac{2\ R0}{Rs} = (S\ g_e)^{\frac{2}{3}}\ 2\left[2\ \alpha fs\ \left(\frac{Q}{3}\right)^2\right]^{-\frac{2}{3}}.$$

One can express the mass-energy integral in "normalized coordinates;"

$$U(\text{sphere}) = Mc^2 \int_0^{\infty} \rho Mass1\ du \stackrel{\text{def}}{=} M_{\text{sphere}}c^2 \quad \text{and} \qquad \text{(SI-10)}$$

$$\rho Mass1 = \frac{\gamma}{Iu - \gamma}\left[\frac{f^{-1}}{Iu - \gamma}\left(2 - \frac{3}{f}\right)\left(1 - \frac{2}{f}\right) - 6u^2\right](|g_{11}|\ g_{44})^{0.5}\ .$$

Then it is seen that the integral must evaluate to unity for equality. Expressing the mass energy-density in this fashion incorporates the effect of the distorted volume on the mass-energy's radial dependence. The sphere-distortion energy is thus seen to be a function of "charge energy" through the integration constant γ and to "spin-energy" through the coupling constant κ.

One comprehensively describes such a geometric distortion family therefore by mimicking a physical "mass" quantity and a "spin" quantity, the "charge" quantity being subsumed in the former "spin" quantity. This should arise from the "two-basis-tensor" geometric foundation. The "mass-energy" quantity is further satisfied internal to the model in the geometrostatic tensor construct of equation (SI-6). In summary, equations (SI-4) through (SI-9) form the equation set for describing the distortional family's "particle-like" characteristics and its associated Maxwellian-like fields. Classically and geometrically, any mass quantity is allowed and describable.

Since the distortional description itself yields a constrained value for the geometrostatic coupling constant, we see that at the mass and charge levels considered here, considerably larger values than the gravitational coupling constant are involved. It is to be noted that the gravitational coupling constant, G/c^4, is approximately 37−42 orders of magnitude smaller than these geometrostatic Maxwellian-mimicked coupling-constant values.

Gravitational Field Energy density(units of Joules/meter^3,

$$u_G = \frac{G(Mc^2)^2}{8\pi c^4}\frac{1}{r^4} = \kappa_G \frac{(Mc^2)^2}{8\pi r^4};\ \kappa_G \stackrel{\text{def}}{=} \frac{G}{c^4}\ .$$

Electrostatic Field Energy density

$$u_E = \frac{\varepsilon o}{2}E^2 = \frac{\epsilon o}{2}\left(\frac{q}{4\pi\varepsilon o}\right)^2 \frac{1}{r^4} = 2\ \kappa o \frac{(Mc^2)^2}{8\pi r^4};\quad \kappa o = \frac{1}{8\pi\ \varepsilon o}\frac{(q)^2}{(Mc^2)^2}\ .$$

Magnetostatic Field Energy density,

$$u_B = \frac{1}{2\mu o}B^2 = \frac{\mu o}{2}\left(\frac{\mu_spin}{2\pi}\right)^2 \frac{1}{r^6} = 2\ \kappa o \frac{(Mc^2)^2}{8\pi r^6}\frac{(\hbar c\ S\ g_e)^2}{(Mc^2)^2}\ .$$

Geometrostatic Field Energy density,

$$u_{geo} = -g_{11}g_{44}\left(Td_4^4 + Td_1^1\right) + 2g_{11}g_{11}\left(\frac{Td_4^4 - Td_1^1}{2}\right) =$$

$$= \text{as } r\to\infty \; = \kappa_{geo}\frac{(Mc^2)^2}{4\pi r^4}\left\{1 + \frac{(\hbar c\, S\, g_e)^2}{(Mc^2)^2 r^2}\right\}; \quad \kappa_{geo} = \kappa o + {}^{\kappa_G}/_2$$

The field equations, in both the EM realm and the gravitational realm (Q = 0), exhibit r^{-6} geometric behavior which we have interpreted as constituting a "magnetic monopole" mimic (what is a "magnetic monopole" ?).

A 2-dimensional plot of this structure is included here to help visualize the "distorted-geometry" model.

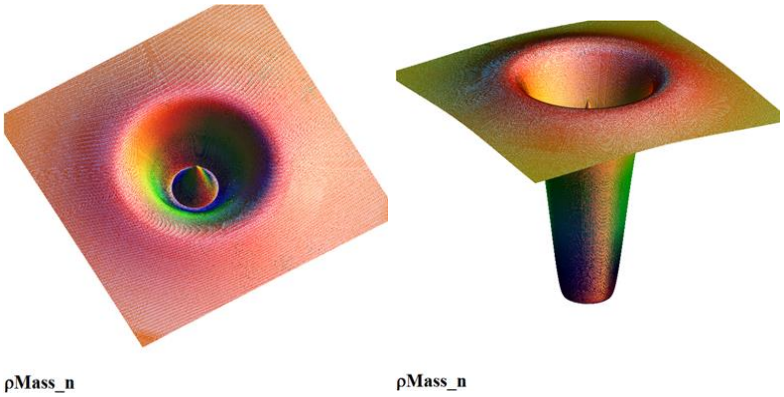

Figure SI-1 Mass-Energy-Density distribution-function surface-plots (two views) (linear radii and logarithmic amplitudes) for the geometric hole distortion.

References

[SI-1 Tolman, R. *Relativity, Thermodynamics and Cosmology*. Dover, NY, 1987 248 .
[SI-2] Koehler, D. Geometric-Distortions and Physical Structure Modeling. *Indian J. Phys*., 2013 **87**, 1029 .
[SI-3**]** Linder, E. V. First Principles of Cosmology; Addison Wesley: Essex, England, 1997; p23
[SI-4**]** Reissner, H. Ann. Phys. 1916, 59 106
[SI-5**]** Nordström, G. Proc. Kon. Ned. Akad. Wet. 1918, 20 1238

SUPPLEMENTARY VIDEO

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As a supplementary visualizing addition to the geometric modeling we include as a Supplementary Video an animated muon video file, simulating the muon to electron beta-decay, a higher-energy nuclear process), produced as a spherically symmetric representation with the following details:
frames 0-15; geometric-distortion mass-energy-density function for muon,

@ frame 15, “muon” transitions (morphs) to “W^-boson”, and

@ frame 30-45, “W^-boson” transitions (morphs) to “electron + neutrinos”;
neutrinos not displayed.

The beta-decay animation is constructed with linear radii but with logarithmic amplitudes and logarithmic normalizing radii R0 and is further normalized to a “neutrino” amplitude and an “electron” radius.